# QUANTUM PROCESSES AND THE FOUNDATION OF RELATIONAL THEORIES OF SPACE AND TIME

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**Abstract**

We present current theories about the structure of space and time, where the building blocks are some fundamental entities (yes-no experiment, quantum processes, spin net-work, preparticles) that do not presuppose the existence of space and time. The relations among these objects are the base for a pregeometry of discrete character, the continuous limit of which gives rise to the physical properties of the space and time.

## 1   Introduction

The theory about the structure of space and time has been discussed from very long time not only among philosophers but also among physicists. It is very well known the correspondence between Newton's disciple S. Clark and Leibniz about the absolute or relational character of space and time [1]. The laws of mechanics and electromagnetism were written with the hypothesis of an absolute space and time but relational theories were unable to construct a consistent model of physical world and were abandoned. With the invention of the special theory of relativity the old arguments of relational theories were renewed but only in the phenomenological level. Einstein himself recognized the atractive ideas of Leibniz as a pioneering work in the theory of relativity [2].

In this century the controversy among the physicists who defend the absolute or relational theory of space and time has increased in great amount from different reasons [3]. The experimental and theoretical development of general relativity and cosmological models have forced to review the concepts of space and time. The revision of the foundations of quantum mechanics has moved some physicists to reduce the structure of space and time to some properties of quantum processes [4]. Finally, the study of fundamental axioms of geometry (in the mathematical as well in

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the physical sense) has introduced new light into the discussion about the primitive or derived character of space and time.

# 2 Some modern theories about the relational character of space and time

We don't want to go again in the arguments given by the two positions. Instead we will expand some recent relational theories of space and time to prove that the basic assumption common to all of them is that there exists some fundamental objects out of them the structure of space and time is constructed. We will identify those objects as events, processes, yes-no experiments, monads (following Leibniz's ideas) and so on. Therefore the conceps of space and time are for these authors derived concepts in the logical as well in the ontological level.

R. Penrose [5]. His model starts with objects and the interrelations between objects. An object is thus "located" either directionally or positionally in terms of its relations with other objects. "One does not really need a space to begin with. The notion of space comes out as a convenience at the end". The physical objects consist of some units with definite spin acting among themselves giving rise to a N-unit with a total spin representing a direction. The interaction of two N-units es interpreted as the angle between the two directions.

V. Kaplunovsky, M. Weinstein [6]. They consider the notion of space-time continuum as an illusion of low energy dynamics. Quantum systems are defined without the notion of space. The $x_m$ variables have to be replaced by the labels for quantum degrees of freedom. Bose and Fermi fields satisfy the canonical commutation relation, where the quantum degree of freedom play the role of site variable.

$$[\Pi(\ell), \phi(m)] = -i\delta_{\ell m}, \left\{\Psi_\alpha^+(\ell), \Psi_\beta(m)\right\} = \delta_{\alpha\rho}\delta_{\ell m}$$

The hamiltonian incorporates link fields $X_{(\ell m)}$ and $P_{(\ell m)}$ to boson and fermion fields, satisfying

$$\left[P_{(\ell m)}^{\alpha\beta} X_{(\ell' m')}^{\alpha'\beta'}\right] = -i\delta_{\ell\ell'}\delta_{mm'}\delta_{\alpha\alpha'}\delta_{\beta\beta'}$$

The hamiltonian is constructed out of such fields.

$$H = \sum_{p,q} \left[-iX_{(p'p)}\Psi^+(p',q)\sigma_x\Psi(p,q) - iX_{(q'q)}\Psi^+(p,q')\sigma_y\Psi(p,q)\right]$$

All the fields are labelled by integer indexes, that is, they are connected among themselves as a finite network, with the structure of 1-simplex, (where each field is acting with all the other fields) and the different 1-simplices are interacting with other 1-simplices giving rise to N-simplex of more complicated structure. This is called a generalized SLAC lattice, which underlies the continuum space-time.

M. Garcia Sucre [7]. This author, after classifying the relational theories of space and time, proposes a set-theoretical model of space and time in which the primitive concepts are preparticles as the most basic components of a physical system. Using the membership relation of set theory he construct the elementary particles as subsets of the power set of preparticles. Inclusion relations among preparticles and interaction among them gives rise to the temporal and spatial structure of the world out of which reference frames and motions are described.

M. Bunge [8] proposes a relational theory of physical space. The world is constituded by things such as elementary particles and fields and physical systems are composed out of these things. The concept of space is a derived one, therefore these things are not located in the space. These things are acting among themselves, the result of these interactions are called processes or events. Space is nothing more that the collections of these individual things and their relations.

M. Friedman [9] starts from the set of concrete physical events $\mathcal{E}$, or, the set of space-time points that are actually occupied by material objects or processes. There is a relationalist's ontology by which the (3+1)-dimensional continuum is nothing more than a construction out of the set $\mathcal{E}$ of physical events.

Mundy [10] offers a relational theory of Minkowski space-time. Suppose that there are a finite number of point particles and the mutual spatial relations among the particles obey the axioms of euclidean geometry. Any talk about space is to be analyzed according to the representations of the interparticle relations. Of course these theories must recover the field-theoretical models out of the individual events that are finite in number. A real challenge for the relational theories.

J.A. Wheeler [11] has introduced the concept of pregeometry to understand the laws of euclidean and non-euclidean geometry. The objects in the pregeometry are logical in character. Space and time are not physical entities but conceptions created by man, to keep track of the order of things. Time and space must be derived in the classical limit from the universe of our discrete elements (or bits) of information.

Ponzano and Regge [12] have develope Wheeler's ideas in some model of curved space time using spin-network, the elementary structure of which are the $3j$-simbols for 3 particles of spin $1/2$ acting among themselves. The structure of a curve 2-dimensional surface is described by the closed interaction of half-integral spin particles whose diagramatic representation is given by a closed finely triangulated 2-surface with $2n$ triangles and $3n$ edges. The fineness of the diagram, determined by $n$, can be used to derive a correspondence law between the basic pregeometry and the continuous limit of physical geometry. The Ponzano-Regge model has been developped by modern authors in the study of Quantum Gravity.

# 3   An axiomatic formulation of a euclidean lattice

In the last section we have reviewed some modern theories in which the concepts of space and time are derived from the relations among fundamental objects (building blocks). The underlying structure should be compared with the properties of the physical space and time. We want to develope now some pregeometry in which the axioms are given by mutual relations of these building blocks.

Given some reticular network of fundamental objects we can postulate from purely logical properties (in the sense of Wheeler's pregeometry) a N-dimensional cubic lattice. The difference between Hilbert's axiomatic formulation and our approach is the following: HILBERT [13] presuposes the concepts of points, lines and surfaces, and the axioms are constructed with these objects. In our approach the points, lines and surfaces are derived from the relational character of the lattice, but the logical consecuences of the lattice are equivalent to the set of axioms in Hilbert's formulation.

For our approach we need only one Axiom for each particular lattice.

**Axiom of tesselation:** Given an indefinite number of objects, each of them is connected with no less and no more than $2N$ different ones, forming a $N$-dimensional lattice.

This Axiom is called of tesselation (or saturation) because for each dimension the points are completely connected and don't admit new connexions.

From this Axiom we can derive the definition of straight line, principal straight lines, orthogonal and paralell straight lines. [14]

A *path* is the connection between two different points, say, A and B, through points that are pairwise neighbours.

The *length* of a path is the numbers of points contained in the path, including the first and the last one.

A *minimal path* is a path with minimal length (in the picture the two paths between A and B are minimal). Between two point there can be different minimal paths.
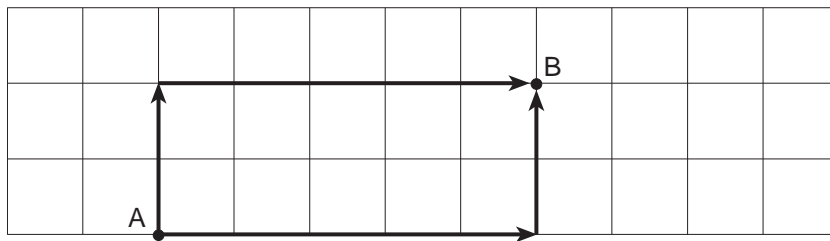


A *principal straight line* is a indefinite set of points in the lattice, such that each of them is contiguous to other two, and the minimal path between two arbitrary points of this line is always unique.

**Theorem 1**: through a point of a 2-dimensional square lattice pass only two different principal straight lines (they are called *orthogonal straight lines*).

**Theorem 2**: two principal straight lines that are not orthogonal have all the points either in common or separated (in the last case they are called *paralell straight lines*)

From these two theorems we can define cartesian (discrete) coordinates and an euclidean space. This structure of 2-dimensional space can be easily generalized to 3-dimensional cubic lattice. With the help of cartesian coordinates we can define a (non principal) straight line passing through two arbitrary points.

It is possible now to deduce from the Axiom of tesselation the properties of the lattice that are given as Axioms in Hilbert's geometry. For the 2-dimensional lattice we have:

1. There is a unique straight line through any two different points. Any straight line contains at least two points. There are three non-colinear points.

2. Given three points A,B,C of a straight line if B is between A and C, A,B,C are three distinct points, and B is between C and A. Given two points A and C, there exists one point B over the straight line AC such that C is between A and B. Given three points on a straight line there is only one between the other two.

3. Given two points A,B of a straight line, and a point A′ of the same or different line there exists a point B′ in the same line as A′ such that the segment A′B′ is congruent with AB. If two segments are congruent with a third one, they are congruent one to another.

4. Given a straight line and a point exterior to it, there exists one and only one straight line that incides in the point and does not cross the first one.

For more complicated lattice we can follow the "skeleton" geometry of Wheeler used to describe the topological properties of curved space-time, that where developped by Regge and recently by many other authors. [15]

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